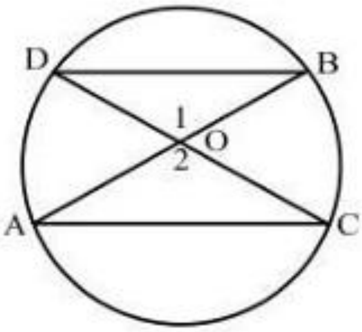
PROPORTIONALITY AND SIMILARITYAturday 2nd May 2026

EXAMPLE 9

A, B, C and D are concyclic points. DOC and AOB are chords. DB and AC are joined. Prove that:

(a) $\Delta AOC ||| \Delta DOB$

(b) $\frac{OB}{OD} = \frac{OC}{OA}$



Statement	Reason
(a) Match the corresponding angles of ΔAOC and ΔDOB as follows and then prove the pairs of angles equal.	
$\hat{A}$ —— $\hat{D}$ $\hat{O}_2$ ········ $\hat{O}_1$ $\hat{C}$ —— $\hat{B}$ Draw solid lines for each pair of corresponding angles that are equal. The dotted line indicates that the pair of angles are equal due to the sum of the angles of a triangle.	
In ΔAOC and ΔDOB:	
(1) $\hat{A} = \hat{D}$	arc BC subtends equal angles
(2) $\hat{C} = \hat{B}$	arc AD subtends equal angles
(3) $\hat{O}_2 = \hat{O}_1$	sum of angles of Δ
$\therefore \Delta AOC ||| \Delta DOB$	$\angle, \angle, \angle$
Note: You could have also used the reason "vertically opposite angles" for statement (3) above.	
(b) $\Delta AOC ||| \Delta DOB$	
$\therefore \frac{AO}{DO} = \frac{OC}{OB} = \frac{AC}{DB}$	corr sides of Δ's in proportion
$\therefore \frac{OA}{OD} = \frac{OC}{OB}$	
$\therefore \frac{OB}{OD} = \frac{OC}{OA}$	cross multiplication

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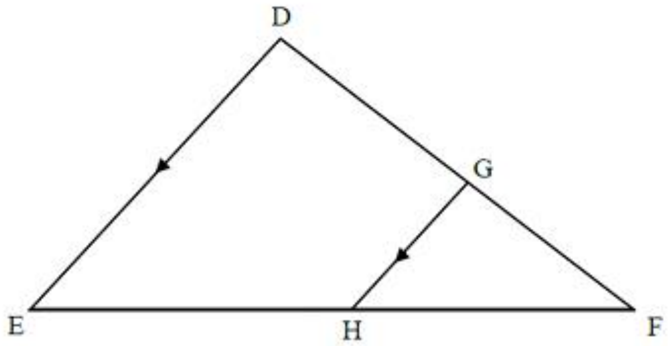
QUESTION 9

9.1 In the diagram, ΔDEF is drawn. Line GH intersects DF and EF at G and H respectively such that $GH \parallel DE$ and $\frac{GF}{DG} = \frac{2}{5}$.



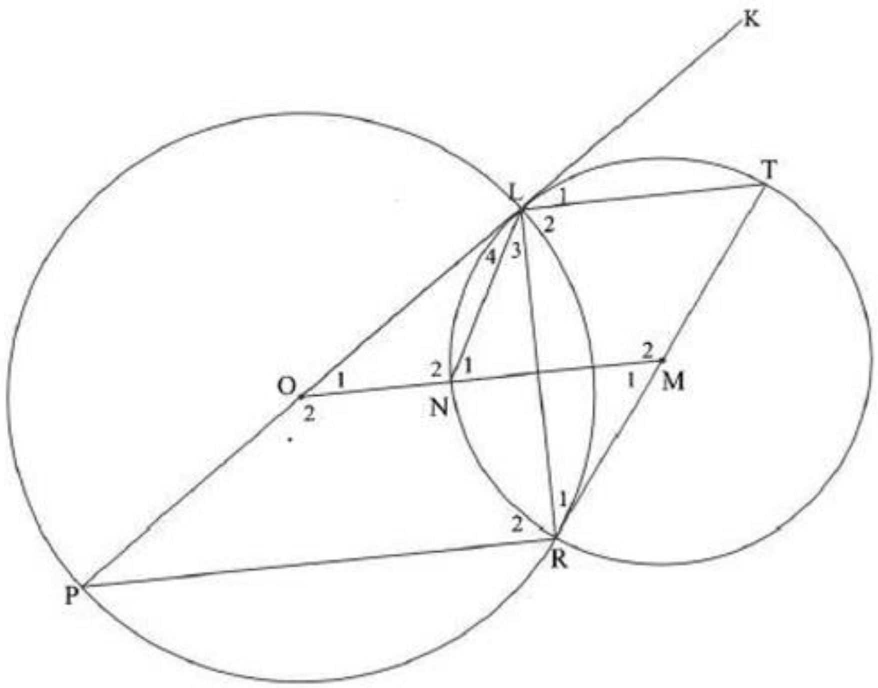
9.1.1 Write down, with a reason, the value of $\frac{HF}{EH}$. (2)

9.1.2 If $EF = 21$ cm, calculate the length of EH. (2)

9.1.3 Write down a triangle which is similar to ΔFGH. (1)

9.1.4 Hence, calculate the value of $\frac{GH}{DE}$. (2)

9.2 In the diagram, POL is a diameter of the larger circle with centre O. TMR is a diameter of the smaller circle with centre M. The two circles intersect at L and R. PLK is a tangent to the smaller circle at L and TR is a tangent to the larger circle at R. OM intersects the smaller circle at N. Straight lines LT, LR, LN and PR are drawn.



Prove, giving reasons, that:

9.2.1 LT || PR (4)

9.2.2 LORM is a cyclic quadrilateral, if it is also given that LT || OM (5)

9.2.3 LN bisects $O\hat{L}R$ (4)

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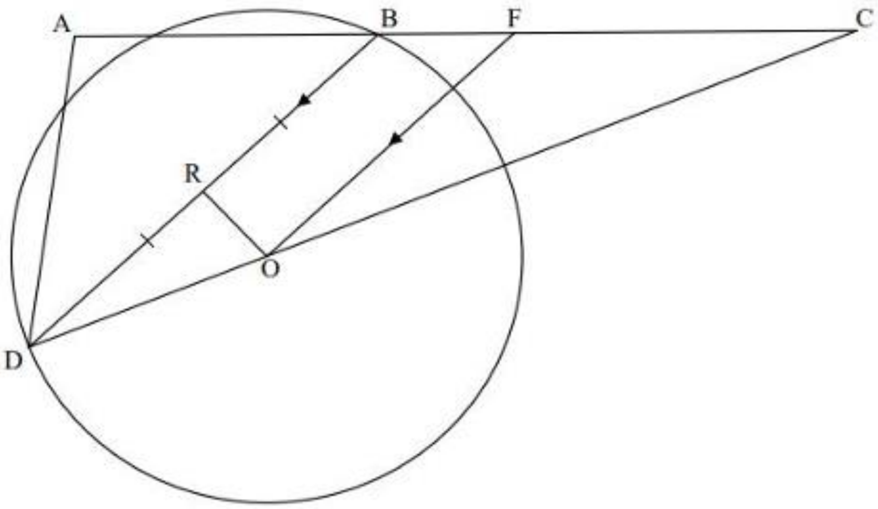
10.2 In the diagram, O is the centre of the circle. Points D and B lie on the circle. Points A and C lie outside the circle such that side AC of ΔADC passes through B. F is a point on BC such that $FO \parallel BD$. $DR = RB$ and RO is drawn.



10.2.1 Prove, with reasons, that $\Delta CFO \,|||\, \Delta CBD$. (3)

10.2.2 If it is given that $R\hat{D}O = F\hat{C}O$, show, with reasons, that $OF.CD = CO.BC$ (2)

10.2.3 It is further given that $DC = 19{,}2$ units, $BD = 12$ units and $\frac{RO}{RD} = \frac{3}{4}$

Prove, with reasons, that $BF = \frac{75}{16}$ (6)

10.2.4 Calculate the size of $A\hat{B}D$. (3)

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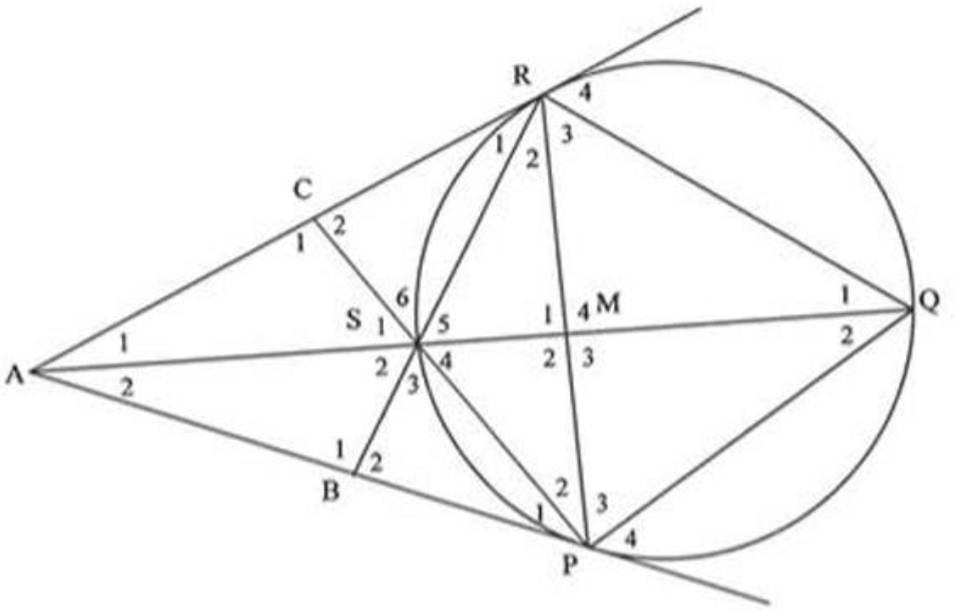
QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.1 $\hat{S}_3 = \hat{S}_4$ (5)

10.2 SMRC is a cyclic quadrilateral (4)

10.3 RP is a tangent to the circle passing through P, S and A at P (6)

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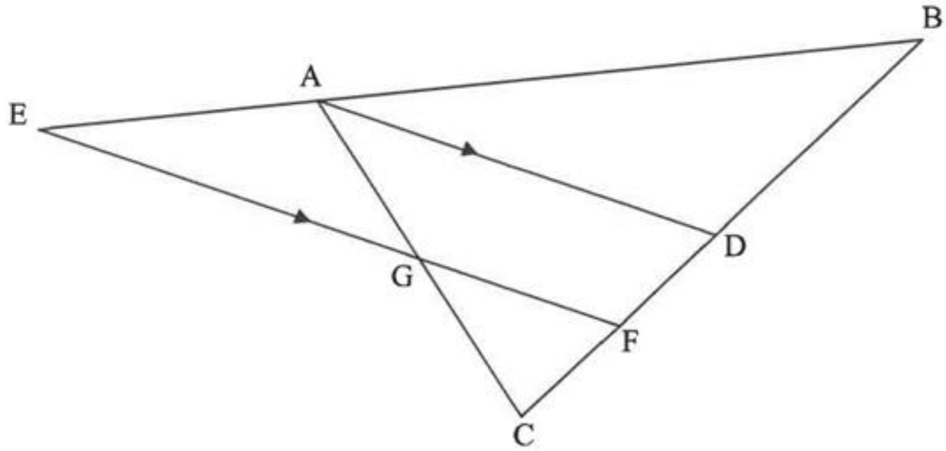
QUESTION 11

11.1 In the diagram, ΔABC is drawn. BA is produced to E. F and D are points on BC such that $AD \parallel EF$. AC and EF intersect at G. $\frac{CF}{FB} = \frac{2}{5}$ and $\frac{CG}{GA} = \frac{3}{2}$.



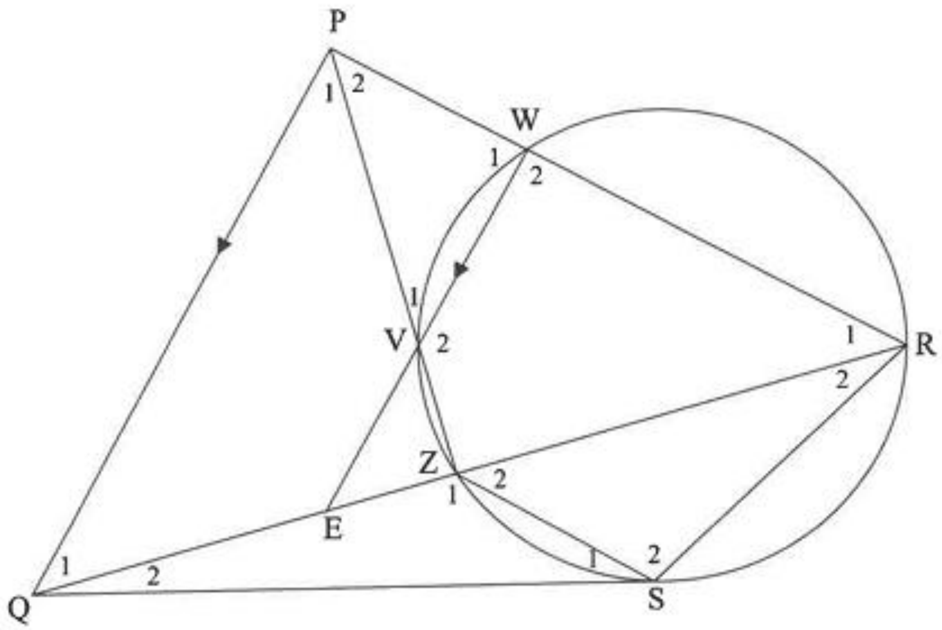
Calculate, with reasons, the value of:

11.1.1 $\frac{FD}{CF}$ (2)

11.1.2 $\frac{BA}{EA}$ (4)

11.1.3 $\frac{\text{Area of } \Delta GCF}{\text{Area of } GFDA}$ (4)

11.2 In the diagram, WVZR is a cyclic quadrilateral. RZ is produced to Q. A tangent is drawn from Q to touch the circle at S. WV is produced to E, a point on ZQ. RW produced meets ZV produced in P. PQ || WE. RS and ZS are drawn.



Prove, with reasons, that:

11.2.1 $PR = \frac{PW.QR}{QE}$ (2)

11.2.2 If $\Delta PQZ \,|||\, \Delta RQP$, then $PQ^2 = RQ.QZ$ (1)

11.2.3 $\Delta QSZ \,|||\, \Delta QRS$ (3)

11.2.4 $PQ = QS$ (3)

11.2.5 $PW = \frac{QE.PZ}{\sqrt{QR.QZ}}$ (4)

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